Fun with Tops, Part 1

by George Bell

Figure 1. *1 Pinko Ringo* (left), 2 of the 10 pieces (center), *Exploding Ball* (right).

"Just because something doesn't do what you planned it to do doesn't mean it's useless." –Thomas Edison

Introduction

In 2008 Stephen Chin showed me his 10 identical piece ball puzzle (Figure 1, left). Give it a spin, and it spins for a few seconds and suddenly explodes into 10 pieces [1]. The effect is so unexpected that the explosion still makes me laugh every time I see it. Stephen made these puzzles out of wood, he asked me if I could 3D print them. I made a version I called the *Exploding Ball* [1] (Figure 1, right). The only change I made was to include an 11th piece, an icosahedron die in the centre.

I continued to look for physical mechanisms which would allow a puzzle to spin for a while before coming apart in pieces. Clearly this will not be a difficult puzzle, the real challenge is understanding how it works. *1 Pinko Ringo* is a 10 identical piece coordinate motion puzzle which comes apart when each piece moves away from the axis of symmetry. The trick for a delayed explosion is to start it spinning with the axis of symmetry horizontal, then it can’t come apart until the axis of symmetry wanders near vertical. In order for this wandering to occur it is important that the final shape of the puzzle be close to spherical. If the external shape is an icosahedron (the original shape) it tends to get hung up spinning on a vertex.

One promising physical mechanism involves the *Tippe Top* [2]. In Part 1, I will describe the physical principles behind the *Tippe Top*, and a new type of top I created. Part 2 will discuss my efforts to turn this mechanism into a puzzle.

The *Tippe Top*

Woven into our mechanical puzzle solving skills is a simple physical principle, whether we understand it intuitively or explicitly:

> *Any rigid object[1], if it can so move, will move to lower its center of mass (CM).*

This principle explains why cubes (and almost any polyhedron) on a table prefer to lie on a face. If we tilt them upward along an edge, this raises their center of mass (CM). When we release the cube, it falls back to rest on a face. A cube resting on a face is in what in physics is known as a stable equilibrium. If the cube is tilted a restoring force pushes it back towards the equilibrium position.

It therefore comes as a shock to find an object which does not obey this principle. This object is called a *Tippe Top* [2] (Figure 2), it was discovered more than 100 years ago, and the above footnote was added because of it.

Figure 2. An aluminium *Tippe Top* (left), 3D printed *Tippe Tops* [3] (right).

The *Tippe Top* has the appearance of a hollowed half-sphere with an axle (Figure 2). When you grab a *Tippe Top* by the axle and spin it, it flips over, eventually spinning on the axle. As it inverts, its CM actually moves upward. Incredibly, as long as a *Tippe Top* is spinning sufficiently fast, it is unstable when its CM is as low as possible, and stable when its CM is as high as possible.

Figure 3 shows the *Tippe Top* “life cycle”. In position **a** the stationary *Tippe Top* sits on a table, its CM happily as low as possible. In position **b**, someone has given the top a good spin (denoted by the red arrow), for a *Tippe Top* this is an unstable position. It moves away from this vertical position (it falls over), and eventually assumes a “flat spin” (position **c**), for one instant it is no longer spinning around its axle. The transition from **c** to **d** is not explained by this stability analysis, but once the top reaches position **d** it is again stable, spinning with its CM as high as possible.

Eventually, the top is not spinning sufficiently fast and finds itself again unstable (**e**), it topples over and finishes in a much slower flat spin (**f**). As the top’s spin ceases it tips up into the vertical position (**a**), its preferred orientation as a stationary object. We note

[1] Which is not spinning.

that from an observer's perspective the top is always spinning in the same direction, but in the reference frame of the top the direction of rotation reverses [4].

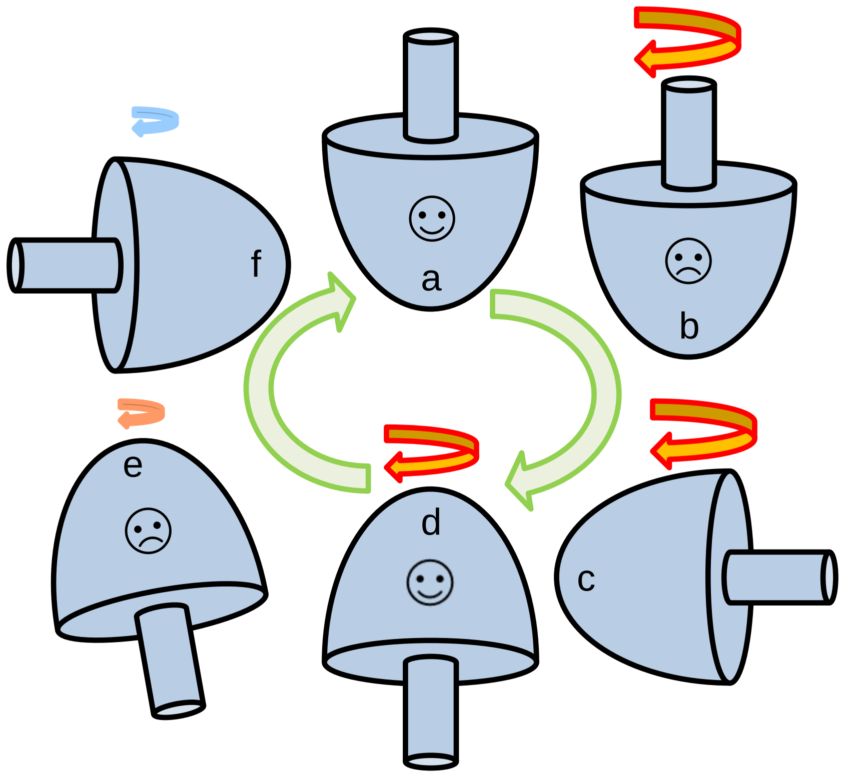

Figure 3. Life cycle of a *Tippe Top*. A happy face indicates the top is stable, a sad face means it is unstable.

One should not jump to the conclusion that all spinning objects are stable with their CM as high as possible. The weight distribution of the *Tippe Top* is also critical in determining its behavior, this can be quantified by the top's principal moments of inertia. Fortunately, we need not delve too deeply into the details because of several recent papers [5, 6].

These papers consider an idealized model of a *Tippe Top* which is spherical in shape with a non-uniform (arbitrary) mass distribution, but symmetrical about the main rotation axis (the **z** axis). This top is defined by several descriptive parameters: size, weight, location of the CM and the principal moments of inertia. In general, all of these descriptive parameters are easy to determine except for the principal moments of inertia.

In [5, 6] the dynamical behavior of a spherical top is partitioned into three Groups. Any particular top is in exactly one Group, depending on its descriptive parameters. What they call "Group II" is the "classical *Tippe Top*", where position **b** is unstable and position **d** is stable (one can think of this as the definition of Group II). Any Group II top must be spinning sufficiently fast to show this special behavior. We can determine which

Group any spherical top lies in by inserting the descriptive parameters into formulas derived in [5, 6]. Other formulas also quantify the meaning of "sufficiently fast".

The *Flippe Top*

Imagine a uniform density (wood) sphere of radius **R** through which a hole is drilled of radius **r**. We insert a steel bearing ball (radius **r**) and add a pair of stops so that the steel ball can move a maximum of **d** from the centre of the large sphere. This object is diagrammed in Figure 4. At this point there are only three design parameters: **R**, **r** and **d**, although the density of the materials used is also important.

Let us assume that we can somehow find **R**, **r** and **d** so that this top behaves like a *Tippe Top* (lies in Group II). What will happen when we spin it? The top is unstable in the position of Figure 4 with the CM low, so it will invert to reach a stable position. When this happens the steel ball will (presumably) drop down, it is then in the initial state except that the top has flipped over. This process will then repeat—this is a *Tippe Top* which flips repeatedly. I call it a *Flippe Top* [7].

One way to make a *Flippe Top* is using 3D printing. It is convenient to print it in two halves with different colours (making it more obvious when a flip is executed). In FDM (fused deposition modelling) 3D printing, normally a lower density is used for the interior of an object (to save material as well as time). In [7] I calculate the descriptive parameters assuming a reduced interior density. My standard size top has diameter 5 cm (~2") and contains a steel ball of diameter 12.7 mm (1/2"). For these parameters, I found that **d** should be between 6 mm and 9 mm to hit the Tippe sweet spot (Group IIa or IIc) [7]. This narrow range may explain why this flipping top does not appear to have been observed before.

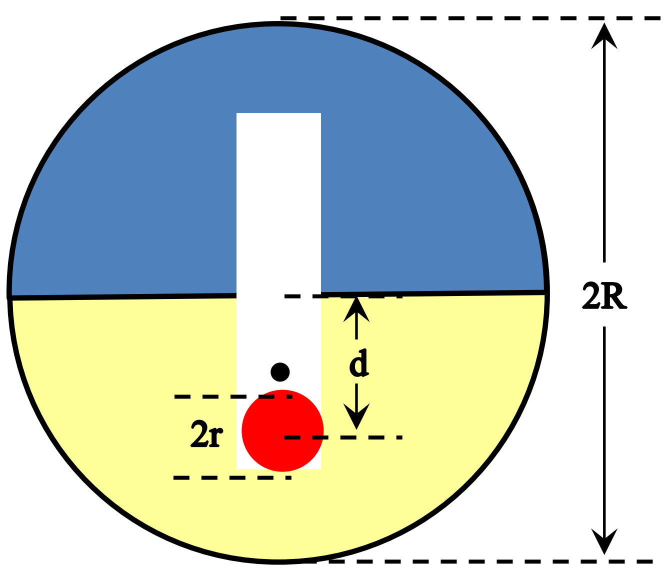

Figure 4. Schematic of a *Flippe Top*.

If you make a *Flippe Top*, one important detail is that the cylindrical hole must pass through the entire top, see [7]. In addition to the standard size, I scaled this *Flippe Top* up by a factor of 5/4 and down by a factor of 3/4. The three sizes are shown in Figure 5. All behave similarly. You can print out your own copy at [8]. I put videos of these tops on YouTube [9].

Spinning by hand on a plate, it is fun to see how many flips you can get out of a *Flippe Top*. It is not hard to get 5 flips and my record is 7. The smallest size seems to work best for this. Every time the top flips, some of its angular velocity is converted into the energy of lifting a steel ball by *2d*. One can calculate the maximum number of flips a top can execute given an initial spinning rate and perfect energy conversion [7]. The *Flippe Top* mechanism can be viewed as a braking mechanism for the top, it will spin longer if the steel ball is fixed and not allowed to move.

Figure 5. Three sizes of *Flippe Top*: diameter 37.5 mm, 50 mm and 62.5 mm. The largest size has an equatorial groove for a string.

There is a limit to how fast these tops can be spun by hand. I increased the starting spinning rate by the usual top trick of wrapping a string around them. High speed spins show an interesting new phenomenon. The top will invert, but then the steel ball does not drop down. My explanation for this is the following: all spinning objects exhibit a motion called precession. These tops precess at a high frequency and when doing so, a component of the precession velocity pushes the steel ball outward, holding it at the top of the cylinder. This holds the steel ball until the top slows, whereby the steel ball can drop down, and the flipping begins.

Summary

We have considered a spherical top containing a steel ball which can move up and down in a cylindrical channel. If the parameters of this top are chosen carefully, it behaves like a *Tippe Top* that resets every time the steel ball falls down the channel. Because it flips multiple times, I call it a *Flippe Top*.

All *Tippe Tops* work because their principal moments of inertia have exactly the right properties. One of the reasons their behaviour is counter-intuitive is that the principal moments of inertia of any object are not easy to comprehend, or even determine. If you want to find the mass of the metal *Tippe Top* in Figure 2, you can determine it to a tenth of a gram using a postage scale. If you want to find the principal moments of inertia to high precision, you will find this difficult. Yet the moments of inertia are more important than mass in determining what happens when the top spins.

In Part 2 we will see how to use the *Flippe Top* mechanism in a puzzle.

References

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