Fun with Tops, Part 2

by George Bell

"Our greatest weakness lies in giving up. The most certain way to succeed is always to try just one more time." –Thomas Edison

Introduction

In Part 1 of this article series [1], I introduced a new top called a *Flippe Top*. It is a *Tippe Top* in the shape of a sphere with a steel bearing ball inside. The steel ball is free to move within a cylindrical channel. When the top is spun, it behaves like a *Tippe Top* and inverts. After it inverts the steel ball drops down the channel and the cycle repeats, flipping over and over. My *Flippe Top* is now sold by Grand Illusions [2].

In Part 2 our goal is to turn this into a puzzle which spins for a while but then falls apart. It would also be nice if there was no other way to open it, or at least no easier way.

How can we make a *Flippe Top* into a puzzle? Imagine a puzzle in the shape of a sphere which comes apart into several pieces. Cut a cylindrical channel inside and add a steel ball (Figure 1). Suppose we can arrange things so that the steel ball holds the puzzle together in position A, but allows it to fall apart in positions B or C. Then, when spun with A down, the top will hold together. Lastly, imagine that we can arrange the principal moments of inertia so that this behaves like a *Tippe Top*. Since it is a *Tippe Top* it will invert and the ball will drop to C, causing it to fall apart.

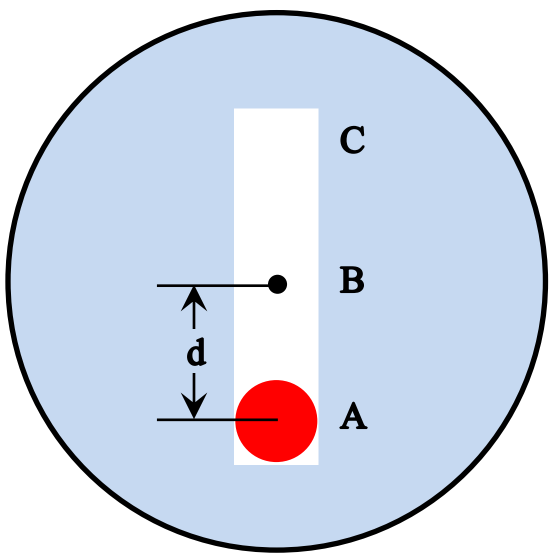

Figure 1. Schematic of the *Flippe Ball* puzzle.

We've made a lot of assumptions, but it turns out that this puzzle can actually be created, and we'll call it the *Flippe Ball* puzzle. There are many coordinate motion dissections of the sphere, each is a candidate for this puzzle. Three-piece dissections seem to work best, with the cylindrical channel aligned with the axis of symmetry.

I now present three versions of the *Flippe Ball* puzzle, each based on a different 3-piece sphere dissection. Although these puzzles are presented as a logical progression, in fact their development was anything but straightforward. I have a box filled with dozens of failed prototypes, and most of my ideas didn't work out. My "Version 0" was actually the inspiration behind the original idea of hollowing out the cylindrical core. Version 0 was actually invented before the *Flippe Top*.

Version 0

In 2019 I discovered several coordinate motion dissections of the triacontahedron into 3 or 4 pieces [3]. These can be modified into sphere dissections by inscribing a sphere in the triacontahedron.

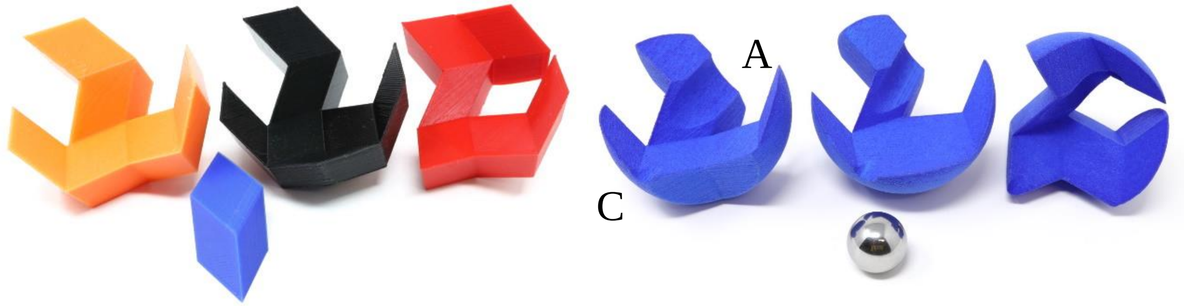

Figure 2. *Swan Puzzle*: triacontahedron dissection (left) and *Flippe Ball Version 0* (right).

This puzzle was originally called GBRT04 [3], but I now call it the *Swan Puzzle* due to the basic piece shape (Figure 2, left). The three main pieces are all slightly different. The fourth blue "rhomb" piece ends up in the core of the puzzle and is optional. The three main pieces go together by coordinate motion; if the rhomb is glued to any main piece it blocks the coordinate motion. It was the channel created by the rhomb that suggested to me further hollowing out the centre.

The blue rhomb can be replaced by a steel ball of the same diameter. If we inscribe a sphere and hollow out an internal cylinder, we get the puzzle in Figure 2, right. This is the only version of the *Flippe Ball* puzzle which has a natural scaling for the size of the steel ball. I designed and ordered a copy of this puzzle from Shapeways [4]. The first prototype allowed one to vary the critical parameter **d** (the maximum distance the steel ball can move away from the ball centre) by inserting wood dowels of various lengths.

The Shapeways puzzle actually worked with the best value of **d** being around 10 mm. One surprise was that after inversion the puzzle would happily spin for quite some time with A up before the steel ball dropped. This puzzle was larger than later versions and contains a 5/8" steel ball, it can spin for 20 seconds before falling apart. Often the steel ball didn't drop until the puzzle had nearly stopped spinning, but it always fell apart when the steel ball hit C.

The problem with this as a puzzle is there is an easier way to open it. If one flips it over so that C is down and gives it a spin, it comes apart immediately. I realized it would be an improvement if I could find a sphere dissection where in Figure 1 the puzzle is held together at A or at C and only comes apart at B. Initially, I didn't realize there was a dissection where this was possible.

Version 1

At IPP39 I played with the *Venn Puzzle* [5] and realized it might be a good candidate to convert into a *Flippe Ball*. I believe the *Venn Puzzle* is a spherical version of Viktor Genel's coordinate motion puzzle *Recube* [6, 7]. Recube was designed in 2011, the details can be found on Viktor Genel's blog [6] (Figure 3).

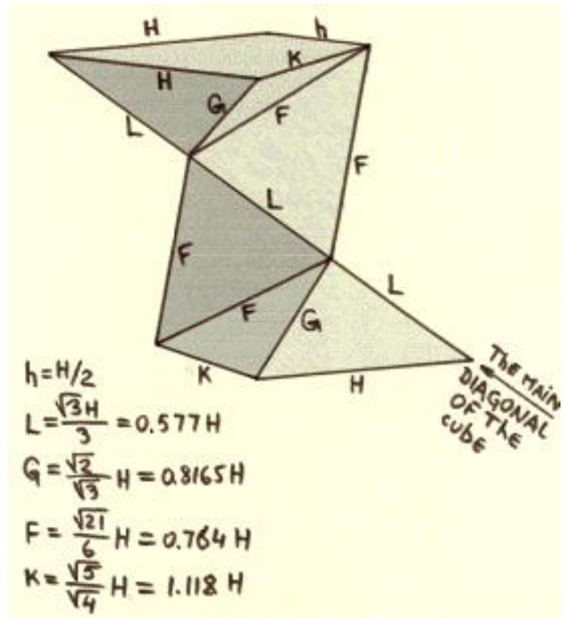

Figure 3. *Recube* original design document (from [6]).

In the units of Figure 3, a sphere inscribed in the cube has diameter H, but this does not result in the *Venn Puzzle*. If the cutting planes are extended beyond the cube, the sphere can be increased to diameter $\sqrt{2}H$ and then appears identical to the *Venn Puzzle*. I removed an inner cylinder of sufficient length so that the puzzle is held together when the steel ball is at the ends of this channel but not in the middle.

A prototype was ordered from Shapeways [4] and it was a failure. The top would invert but the steel ball did not drop down—at least, not right away. As the spinning stopped the steel ball would drop from A to C but the puzzle did not come apart because it was spinning too slowly. Ironically, the most obvious part of the entire mechanism (steel ball falling down a hollow vertical cylinder) did not work. This is not the same problem described in Part 1, it occurs at much lower spin rates.

I discovered that by taping the ball together the steel ball did drop immediately, this essentially converts it into a *Flippe Top*. Eventually I realized that centrifugal force was attempting to fling the pieces outward, and the only thing stopping them was the steel ball (in position A). There was a friction force on each piece preventing this outward motion, and the opposing force of all three pieces on the steel ball was preventing it from dropping down the channel.

It appeared that this prototype was another dead end. Around this time, I purchased an FDM (fused deposition modeling) 3D printer. Printing the curved pieces on an FDM printer was much more challenging than using Shapeways (SLS printer). I printed each piece in two halves, first I tried gluing the halves together but more precise alignment was obtained using tiny steel guide pins.

I had no reason to expect anything different, but when I spun my FDM prototypes (Figure 4, right) they worked! What was wrong with the Shapeways version? After lengthy consideration and testing I concluded that the important difference was the weight of the pieces. The Shapeways pieces were solid and weighed more than 20 g each, while the FDM pieces were hollow and weighed under 13 g. This weight reduction was enough to reduce the friction and make the puzzle work. As the spinning slowed the steel ball would pass B at just the right time so that the pieces would be flung apart. I then printed a hollow version on Shapeways and it also worked (Figure 4, left).

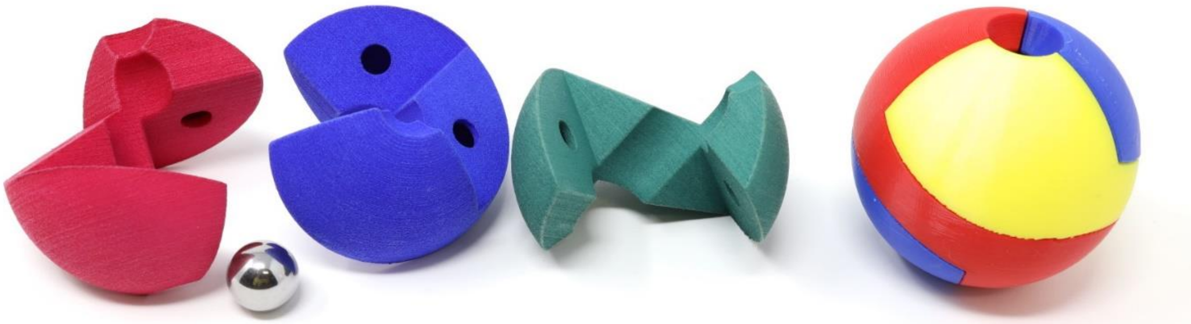

Figure 4. *Flippe Ball* version 1, 3D printed using SLS (Shapeways) and FDM.

The most difficult aspect of these designs is determining how to set the parameter **d**, the maximum distance the steel ball can move away from the centre of the large ball (see Figure 1). With the *Flippe Top*, I was able to calculate its principal moments of inertia [8]; using these parameters and some recent papers [9, 10] I determined optimal values for **d**. This process is very sensitive to the design details—for example removing metal alignment pins or thickening walls can alter **d** by a few mm. With all versions of the *Flippe Ball*, I have not been able to calculate the principal moments of inertia of the pieces. Thus, although the *Flippe Top* **d** is a reasonable starting point, the final **d** is determined by trial and error.

What happens if the parameter **d** is not set correctly? If **d** is too small the spinning top will be stable with the center of mass as low as possible; if **d** is too large the top can "hang" at a stable intermediate state [9]. In either case the top will not reliably invert. It is also possible, if the principal moments of inertia do not have the right properties, that the puzzle will not work for any value of **d**.

In addition, *Flippe Ball Version 1* has another factor which constrains **d.** If the channel is too short the puzzle may not be held together when the ball is at the channel end. For a *Flippe Top* (Part 1), **d** should be between 6 and 9 mm, but at these **d** values *Version 1* is not held together. To hold together **d** must be over 11 mm. In the end I discovered that **d** = 13 mm still works (inverts). If **d** is increased beyond this *Version 1* will not reliably invert.

Version 2

I found a three identical piece sphere dissection which solved two problems. First, the pieces can be printed on an FDM printer without supports. Second, internally there is an adjustable cut (of width **w**) which can be reduced in order that the puzzle holds together for a smaller value of **d**.

Flippe Ball Version 2 (Figure 5) is probably the easiest of the three to assemble. Initially, I assumed that if **w** was smaller than the steel ball diameter **2r** the puzzle could not come apart, because the ball could not escape. However, it turns out this assumption is not correct. I printed out a dozen of these tops with slightly different design parameters as well as 3D printing parameters.

Figure 5. *Flippe Ball version 2*: piece, three variations FDM printed.

I then spun each top 50 times. In the end three critical parameters are **d**, **w** and the piece weight. My final design uses **2R** = 2” (5.1 cm), steel ball diameter **2r** = 12.7 mm, **d** = 11 mm, **w** = 11 mm. Another important parameter is the diameter of the hole the steel ball rides in, it must not be too tight or too loose, I use 13.1 mm. Optimal 3D printing parameters were found to be: layer height 0.20 mm, infill 10%, 2 perimeters.

Top Spinning Tips

While designing of these puzzles, I have performed thousands of test spins. It is difficult to initiate a spin without imparting translational velocity, causing them to fly off. My first tip is to spin them on a large plate on a table to keep them in one place. The ideal plate is concave with no outer rim. A plastic plate is recommended because when a steel ball falls out of a Flippe Ball it could chip or crack a ceramic plate.

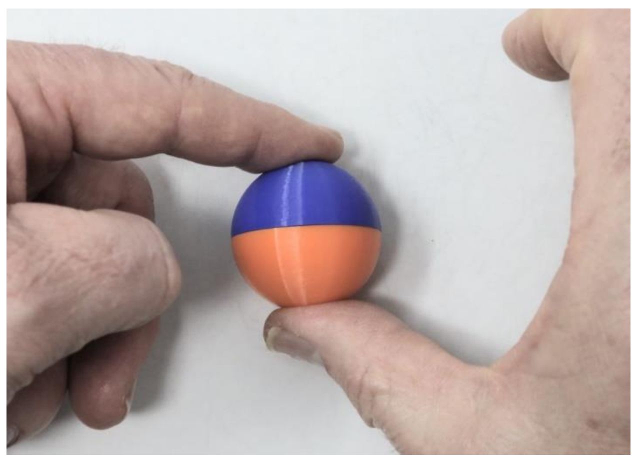

Figure 6. Spinning technique.

My spinning technique is shown in Figure 6. Place your dominant hand (right for me) below the top with your thumb in an indentation. Place your other hand on the opposite side with your index finger against the top. While squeezing the top between these fingers, move your hands rapidly in opposite directions to start the spin. With practice you can initiate a fast spin and the top will stay in place.

A string can also be used for a faster spin. However, faster spins are not necessary and it can be tricky to wrap a string around a sphere.

Summary

I have presented three versions of the *Flippe Ball* puzzle. All spin for a while and then fall apart, exactly our design goal. But is there an easier way to open them? In the case of version 0, yes. Spinning it with C down causes it to fall apart immediately. Versions 1 and 2 can also be taken apart in ways other than spinning. One surprisingly effective method, especially for version 1, is to roll the ball gently across a carpet. A second method is to carefully grab the *Flippe Ball* between thumb and forefinger so that one has a hold of a single piece. If one rocks the puzzle gently back and forth in just the right way it will fall apart, leaving you holding a single piece. This technique is not easy to stumble across randomly.

During my failed prototypes I discovered a way to make these puzzles very difficult to disassemble in any manner. If you taper the channel so it narrows at either end, the steel ball becomes stuck there. To dislodge it you can tap the puzzle against a table, but the steel ball then drops down and becomes stuck at the other end. This is essentially a gravity pin that sticks at either end. I could find no way to open this puzzle other than inserting a small screwdriver to maneuver the steel ball to B.

For *Flippe Ball Versions 1 and 2*, I think that spinning is the easiest way to disassemble them, and certainly the most fun. One interesting aspect of this is that it takes time. Upon spinning on a hard surface, inversion takes a second or two but then perhaps 10 seconds go by before the ball slows sufficiently for the steel ball to drop and the puzzle to fall apart. An impatient solver may assume that spinning isn't working if nothing happens during the first second or two, and grab the ball to halt it.

References

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