The Ball Burr

by George Bell

Recently, I printed copies of my favourite interlocking ball puzzles in the shape of a cube, tetrahedron, and octahedron (Figure 1). *CubeBall* is my design using three non-planar pieces. What I call *Marcus' Tetrahedron* was designed by Markus Götz in 2005 [1]. Markus called it *Curse of the Pharoah*. *Screwy Octahedron* was my exchange puzzle at IPP30 [2].

Figure 1. *CubeBall*, *Marcus' Tetrahedron*, *Screwy Octahedron*.

Each shape in Figure 1 is a subset of the face-centred cubic (FCC) lattice of packed spheres [3]. We can construct each shape by placing a ball at each vertex of the geometric solid. Then add (6, 16, 13) balls in the interior to fill out FCC sphere packing [3]. Other sizes with more or fewer balls are also possible, I simply chose one size for each puzzle.

Figure 2. *Kiss* (assembled into the Ball Burr shape) and *Kissel* (pieces) by Vinco.

Having captured three of the five Platonic solids [4], I wondered about the remaining two—the icosahedron and dodecahedron. I have two ball puzzles by Vaclav Obsivac (Vinco) in the shape of an icosahedron: *Kiss* and *Kissel* (Figure 2) [5]. These puzzles have the same target shape made from 12 balls, which I call a **Ball Burr** (for reasons which will be clear soon).

Kiss and *Kissel* are not related to FCC sphere packing—to create the target shape place a ball at each of the 12 vertices of an icosahedron. Each ball touches 5 others, and there is a void in the centre for a (slightly smaller) ball. This ball (if present) would touch or "kiss" all 12 balls, which gives these puzzles their name.

We should note that Vinco has made five larger ball puzzles based on an icosahedron (*Icosahedron 42, 92, 162, 252, 362*) [5, 6]. These puzzles add additional balls along the edges and faces of the icosahedron (Figure 3). Each size is a different puzzle, having between 6 and 80 pieces.

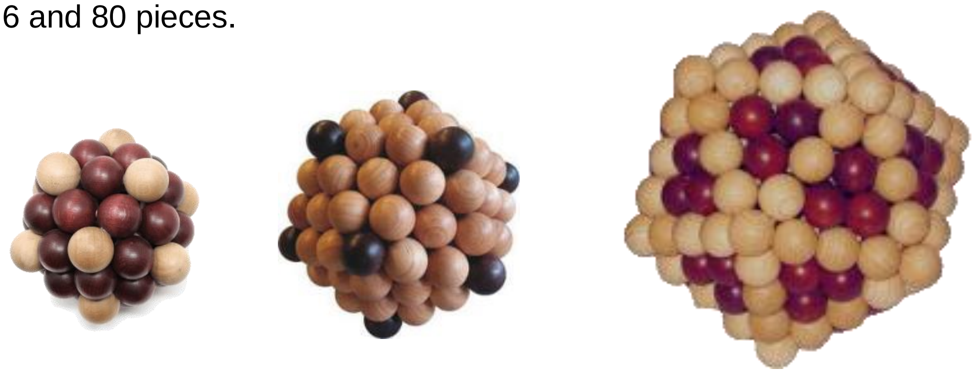

Figure 3. *Icosahedron 42, 92, 162* by Vinco (photos courtesy Vaclav Obsivac).

Puzzle Geometry

I was intrigued by the shape of *Kiss* and *Kissel* (the Ball Burr shape). One way to construct it begins with a group of four balls of diameter 1 as shown in Figure 4. Here $\phi = (1+\sqrt{5})/2 \cong 1.618$ is the golden ratio. Figure 4 shows that this shape is related to the standard *6-Piece Burr*. Although the two shapes are remarkably similar, there are also significant differences.

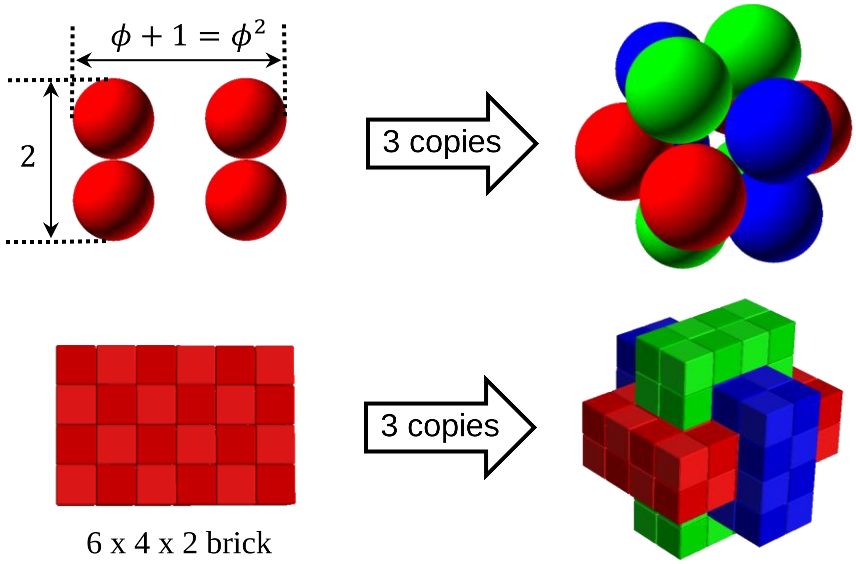

Figure 4. The geometric analogy between the *Ball Burr* and standard *6-Piece Burr*.

One difference is that the Ball Burr shape has additional symmetry. The Ball Burr shape has the symmetry of an icosahedron, it has six 5-fold axes of symmetry and ten 3-fold axes of symmetry. In contrast the 6-piece Burr has zero 5-fold axes and only four 3-fold axes of symmetry.

If we think about the basic puzzle piece, for a *6-Piece Burr* it is a 6x2x2 brick, possibly with some internal voxels removed (Figure 5, top). For the Ball Burr the analogous piece is two balls which do not even touch! We need to connect these balls to create the basic puzzle piece, and there are many options.

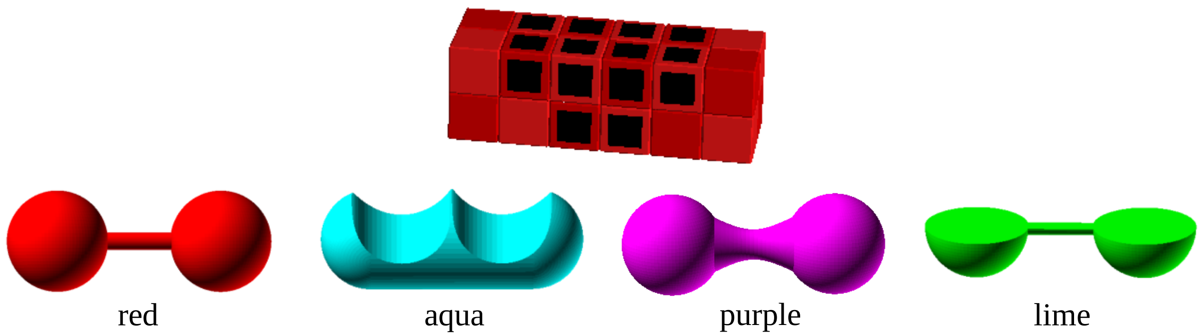

Figure 5. Top: the basic piece for a 6-Piece Burr.
Bottom: options for the basic piece to form the Ball Burr shape.

The first option is to connect the balls using thin dowels which do not interfere with one another or any other balls (Figure 5, red). This is the approach taken by *Kiss* and *Kissel* (Figure 2). The maximum rod diameter is 17.6% of the ball diameter. The most basic puzzle uses six copies of this red piece, we'll call it the *Ball Burr* puzzle. When *Ball Burr* is assembled, no piece is able to move in any direction, each is locked in place by the pieces around it. Each piece touches all five other pieces. Despite geometric similarity, this puzzle does not behave like a 6-piece Burr and *Ball Burr* can only be assembled or disassembled using coordinate motion.

A second option is to connect the balls by a cylinder of the same diameter, some material must then be removed to allow the pieces to fit together (Figure 5, aqua). These pieces behave like standard burr pieces, but made from stock that is round rather than square. The sculptor Charles O. Perry designed *The Ball Puzzle* in 1967 [7], a modern version is *Brass Monkey One* by Ali Morris and Steve Nicholls [8].

Figure 6. *Ball Puzzle* by Charles O. Perry and *Brass Monkey One* by Ali Morris and Steve Nicholls (photos courtesy puzzlemaster.ca).

Charles O. Perry's *Ball Puzzle* and *Brass Monkey One* are standard 6-piece burrs in disguise and can be modelled in BurrTools [9] using 6x2x2 pieces. The Brass Monkey puzzles *Kong*, *Pygmy* and *Marmoset* [8] are more complex burrs in disguise; they can be modelled in BurrTools using 2x2 square stock of length 8 and 10 voxels.

Making Ball Burr Puzzles

The maximum diameter (17.6%) of the connecting rod is quite thin. Made in wood (see Figure 2), these pieces are strong enough, but 3D printed they may be weak and prone to breakage. For certain puzzles we may be able to thicken the connectors, but we need to be careful that our modifications do not interfere in the assembly. This process is different for each puzzle—sometimes the connectors cannot be modified.

3D printing complete spheres is a difficult task, for this reason I cut each piece in half, printed with the flat side down. This strengthens thin connectors because they end up in the bottom few layers, typically printed at 100% infill. Another trick I use for all these puzzles is to add 2 mm diameter holes which fit a short length of filament, these act to align the halves so they can be glued together (Figure 7).

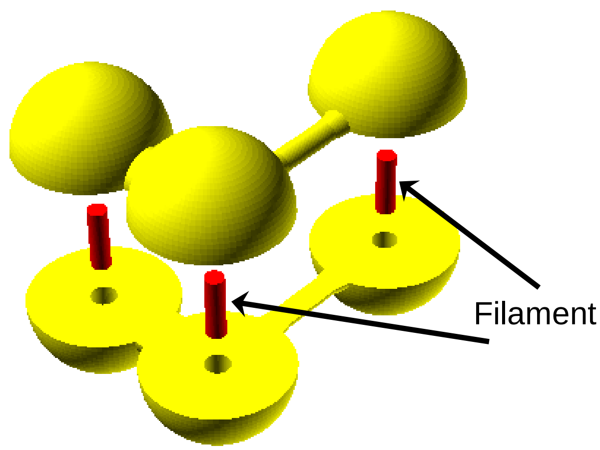

Figure 7. My technique for making pieces using 3D printed components.

New Puzzles

We now investigate new puzzles with the Ball Burr shape. The basic *Ball Burr* puzzle uses six copies of the red piece in Figure 5. *Ball Burr* is similar to Vinco's *Kiss* (which consists of five red pieces plus one special longer piece). *Ball Burr* using thin connectors proves to be a frustrating dexterity puzzle which barely interlocks and needs perfect sizing of both balls and connectors. It falls apart if you touch it. If you have two copies of Vinco's *Kiss* you can "enjoy" this frustrating challenge.

Using thicker connectors (Figure 8, aqua) makes the puzzle much more stable, but assembly is still a dexterity challenge [10]. The aqua piece in Figure 8 uses modified connectors bulked out to 50% of the ball diameter. *Ball Burr* goes together with the same coordinate motion as Stewart Coffin's *Expanding Box* [11]. Assembling *Kiss* is also a dexterity challenge, although the arrangement of pieces is completely different. Both these puzzles can only be assembled or disassembled using coordinate motion.

Interestingly, the *Ball Burr* is closely related to the *Hyperboloid Burr* by Oskar Van Deventer and Naoaki Takashima [8]. The *Hyperboloid Burr* uses something more like the purple piece in Figure 5, where the two halves are connected by a hyperboloid (or several hyperboloids).

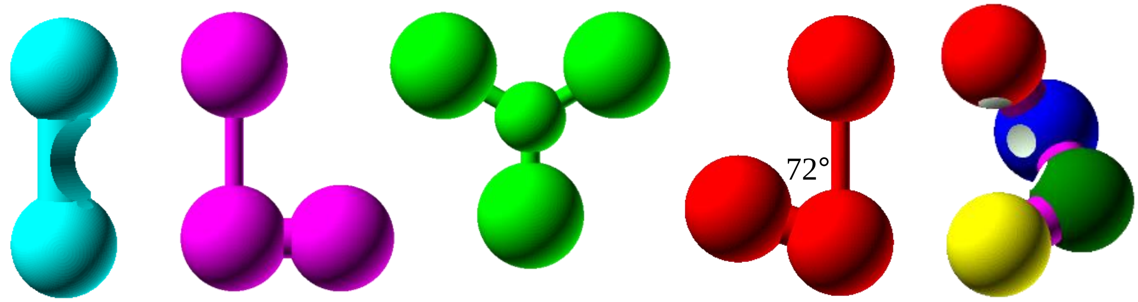

Figure 8. Puzzle pieces: *Ball Burr* piece with a thick connector, L-piece (*Kissel* or *Kiss-LY*), Y-piece (*Kiss-LY*), J-piece, a non-planar (helical) piece.

My new design *Kiss-LY* (aka *LY Icosahedron*) uses three L-shaped pieces plus a new Y-shaped piece (Figure 8, lime). For this puzzle the connectors cannot be thickened because they will interfere with one another. The dimensions of the L and Y-shaped pieces must be as shown in Figure 8 for the puzzle to assemble. The central ball in the Y can be no larger than 67% of the ball diameter. This puzzle reminds me of *YU Octahedron* [2] which uses a similar Y-shaped piece and three U-shaped pieces. But really, the puzzles can't be **that** similar because they assemble into completely different shapes! Figure 9 summarizes these remarkable interlocking puzzles.

Adding the J-piece (Figure 8, red) into the mix would seem to offer many more designs. We can use 4 J's which assemble into the Ball Burr shape, but this puzzle does not interlock. Another puzzle uses 3 J's and one Y, but this puzzle cannot be assembled. The only puzzle I have found which is interlocking and can be assembled uses 3 J's and one L.

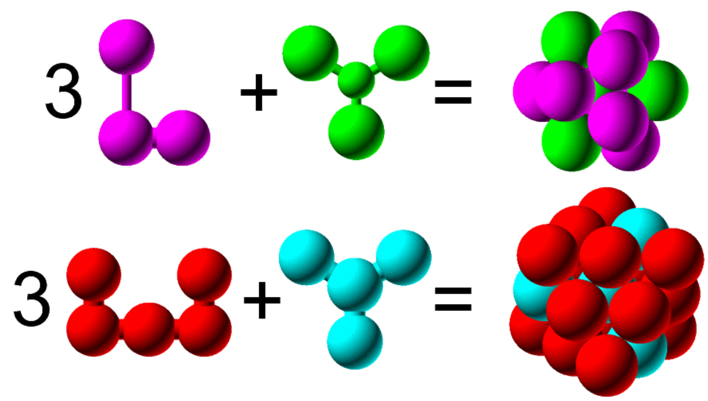

Figure 9. A pair of puzzling equations: *LY Icosahedron* and *YU Octahedron*.

Cutting each ball in half gives many more options for puzzle pieces. This is the lime piece in Figure 5. Using the Figure 4 geometric analogy, I modelled these puzzles in BurrTools [9]. The model uses pieces which are not connected and are not chiral, unlike the real pieces they correspond to. In addition, this model has lost the icosahedral symmetry of the final shape. Most of the puzzles identified by BurrTools are not interlocking or impossible to assemble. I did find one interesting new design, which I call *Diabolical Burr* (Figure 10).

Assembling *Diabolical Burr* does not require dexterity or coordinate motion. The assembly is very loose until the final two pieces are added. The first move to disassemble this burr is unusual and unexpected.

A final puzzle involves three copies of the helical 4-ball piece in Figure 8. The angle between any three connected balls is 108°. Stanislav Knot owns a wood copy of this puzzle (Figure 10) and asked me if I knew the designer. We have been unable to discover the designer or maker. This puzzle does not interlock, and needs a stand for stability. I added a core ball of the same size, and cut "dimples" into each ball (shown in Figure 8) to lock each piece onto the core. My version of this puzzle interlocks, I call it *Ball Icosahedron*. My modification does make the puzzle much easier to assemble.

Figure 10. Left: the seven pieces of *Diabolical Burr* with an assembled puzzle. Right: the puzzle owned by Stanislav Knot on its stand.

Summary

In this article we considered ball puzzles in the shape of an icosahedron, a shape I call a Ball Burr. Although there is a geometric analogy to the standard 6-piece burr, much depends on how the basic puzzle piece is connected.

Name	Shape	# Balls/Pieces		# Planar/Non		Balls/Piece
CubeBall	cube	14	3	0	3	2x5 / 4
Marcus' Tetrahedron	tetrahedron	20	4	0	4	4x5
Screwy Octahedron	octahedron	19	4	1	3	4 / 2x5 / 5
Diabolical Burr	icosahedron	12	7	5	2	3x2 / 4x1½

Table 1. Summary of my favourite interlocking ball puzzles.

Table 1 summarizes my favourite interlocking ball puzzles in the shape of the Platonic solids. One Platonic solid is missing—the dodecahedron. The dodecahedron doesn't seem to work as an interlocking puzzle, see [5] for dodecahedra made by stringing beads.

References

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